

Programmable hash functions

1. The PHF definition we have seen requires that with "decent" probability, $a_{m_i} \neq 0$ for m_1, \dots, m_w , but $a_{m^*_j} = 0$ for m^*_1, \dots, m^*_v . Why can't we expect to have v and w both be large simultaneously?

Such a definition would not be achievable.

Reason why such a definition would not be achievable: let p be the probability (over m , κ and τ) that $a_m = 0$ for a random m . Choose random m_1, \dots, m_k and random m^*_1, \dots, m^*_k . Then $a_{m_i} \neq 0$ and $a_{m^*_i} = 0$ for all i holds with probability $p^k \cdot (1-p)^k \leq (1/2)^{2k}$ even for random κ and τ .

2. A programmable hash function (with "sufficient" programmability parameters)... (choose as many options as you think are appropriate)

- (A) ...is an algebraic tool that should help in enabling a security reduction.
 (B) ...is collision-resistant if the DLog assumption holds in the underlying group.
 (C) ...by definition requires a pairing.

To answer B: Assume the PHF H is $(1, 1, \gamma)$ -programmable. Then we can break DLog in the underlying group with probability $\gamma \cdot \text{Adv}_{\text{CR}}()$, where $\text{Adv}_{\text{CR}}()$ is an adversary against the collision resistance of H .

Proof: Let $h = g^x$ be a DLog instance. Assume the adversary wins the CRHF-game, i.e. he outputs two messages m, m' such that $H(m) = H(m')$ and assume that $a_m = 0$ and $a_{m'} \neq 0$. (The latter happens with probability γ by the $(1, 1, \gamma)$ -programmability of H). Then $h^{a_{m'}} g^{b_{m'}} = g^{b_m} h^{a_{m'}} = g^{b_m - b_{m'}} h = g^{\{(b_m - b_{m'})/a_{m'}\}}$, i.e. $x = (b_m - b_{m'})/a_{m'}$.